Sliding Mode Control with Complementary Inputs for a Lean-Burn IC Engine

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This paper presents a robust controller for an internal combustion (IC) engine, as a first stage of a project aiming at developing a hybrid light urban vehicle, running with ethanol in lean burn.

This work particularly focuses on the design of a sliding mode control for the IC engine of a series hybrid power-train. The controller must allow for speed control and high fuel efficiency.

Simulation results are presented and discussed showing the viability and advantages of the employed control strategy.
Introduction

- The control of internal combustion engines remains an active area for application of advanced control methods. Nowadays, growing interest in hybrid power-trains offers new opportunities for pollution reduction and fuel economy.

- In the current research project, this concept is being explored by a multi-institutional international team focusing on a small series hybrid power-train for a light urban vehicle.

- The engine of the hybrid power train will run on ethanol in lean burn. It will drive a synchronous electric generator, whose voltage is rectified to feed a DC bus. It is well known that a better performance and a simpler design of a controlled rectifier can be obtained when the engine speed remains near its nominal value. Therefore, the primary objectives of this work are speed control and fuel efficiency.
This paper presents a sliding mode design approach for speed control and fuel efficiency of a four-stroke lean-burn operation engine,

which is modelled using a Mean Value Engine Model (MVEM). The engine model used was developed by Crossley and Cook (1991). The original model parameters are kept in this analysis.

It will be later adapted with the actual engine parameters, as soon as the testing stage of the laboratory internal combustion engine is concluded.

The present work is a preliminary approach intended to assess the suitability of sliding mode techniques to control IC engines, with fixed open throttle and lean burn condition, acting only on spark advance ($\theta$) and relative air/fuel ratio ($\lambda$).
Problem

\[
\frac{dx}{dt} = f(x, \lambda, \theta, t)
\]

- Input variables:
  - \(\lambda\) (relative air/fuel ratio) and
  - \(\theta\) (spark advance).

- \(\lambda\) and \(\theta\) belong to a rectangle, however for an optimal performance they have to lie on two of the sides of the rectangle.
\[
\frac{dx}{dt} = f(x, \lambda_{\text{max}}, \theta, t) \\
\frac{d\theta}{dt} = u_1 \\
s = s(x, \lambda_{\text{max}}, \theta, t)
\]  
\[
\frac{dx}{dt} = f(x, \lambda, \theta_{\text{max}}, t) \\
\frac{d\lambda}{dt} = u_2 \\
s = s(x, \lambda, \theta_{\text{max}}, t)
\]
Emptying and filling gas exchange model

Dynamic equation for the pressure $p_i$ (kPa) of the engine intake manifold, assuming constant temperature and volume:

$$\dot{p}_i = \frac{RT_i}{V_i} (-\dot{m}_{at} + \dot{m}_{ap})$$

- $R$ (kJ/kg-K) is the universal gas constant for the fuel-air mixture,
- $T_i$ (K) is the gas manifold temperature, assumed isothermal and constant, and
- $V_i$ (m$^3$) is the manifold volume, assumed constant.

- $\dot{m}_{at}$ (kg/s) and $\dot{m}_{ap}$ are the air mass flow rate crossing the throttle valve and the mixture mass flow rate admitted to the cylinders, respectively.
\( m_{ap} = -0.366 + 0.08979 n p_i - 0.0337 n p_i^2 + 0.0001 n^2 p_i \) with \( n \) (rad/s) the crankshaft rotation speed.

\( m_{at} = f(\alpha)g(p_i) = \left(2.821 - 0.05231\alpha + 0.10299\alpha^2 - 0.00063\alpha^3\right) \left(\frac{2}{p_a}\sqrt{p_ip_a - p_i^2}\right) \)

where \( \alpha \) is the throttle plate angle (90 deg) and \( p_a \) is the atmospheric pressure.
Crankshaft rotation speed model

Dynamic equation for the crankshaft rotation speed, $n$.

$$\dot{n} = \frac{1}{I} (T_e - T_L)$$

- $I$ (kg-m$^2$) is the crankshaft load inertia, $T_e$ (N-m) is the engine torque generation and $T_L$ (N-m) is the load torque.

- The torque generated by the IC engine is represented by a polynomial fit of measured data for different engine operating conditions.

$$T_e = -181.3 + 379.36m_{ap} + 21.91 \left( \frac{A}{F} \right) - 0.85 \left( \frac{A}{F} \right)^2 + 0.26\theta - 0.0028\theta^2 + 0.027n - 0.000107n^2 + 0.00048n\theta + 2.55\theta m_{ap} - 0.05\theta^2m_{ap}$$
\( \frac{A_F}{F} \) (dimensionless) is the mass air-fuel ratio, \( \theta \) (deg) is the spark advance and \( m_{ap} \) (g/s) is the gas mass charged in the cylinder during the intake stroke, which takes place in the first \( \pi \) radians crankshaft rotation of the four-stroke cycle.

\[
m_{ap} = \frac{\dot{m}_{ap}}{n} \pi, \quad \frac{A_F}{F} = \frac{\dot{m}_{ap}}{\dot{m}_f}
\]
where \( \dot{m}_f \) is the fuel mass flow rate flowing to the cylinders, injected by the actuators also in the first \( \pi \) radians crankshaft rotation.

\( \lambda = \frac{\frac{A_F}{F}}{a_s} \) where the constant \( a_s = 14.6 \) is the stoichiometric air-fuel mass ratio. Note that \( \lambda = 1.0 \) at stoichiometry, \( \lambda < 1.0 \) for fuel rich mixtures are \( \lambda > 1.0 \) for fuel lean mixtures.
Operating at the leanest admissible mixture, this implies the maximum relative air/fuel ratio $\lambda$, the engine under study reduces the amount of fuel consumption. Then, if possible, $\lambda = \lambda_{\text{max}}$ and the variation of the load torque is counteracted by appropriately varying the spark advance.

At $\theta_{\text{max}}$ the engine generates its correspondent maximum torque for each constant value of $\lambda$ (in this particular case, $\lambda_{\text{max}}$). Therefore, once $\theta_{\text{max}}$ is reached, that input is kept constant $\theta = \theta_{\text{max}}$ and the load demand must be supplied by controlling the other input $\lambda$. 
\[
\dot{n} = \frac{1}{I} (T_e - T_l) \\
\dot{p}_i = \frac{RT_i}{V_i} (-\dot{m}_{at} + \dot{m}_{ap}) \\
\dot{\theta} = u_1 \\
\lambda = \lambda_{max} \\
\dot{\lambda} = u_2 \\
\theta = \theta_{max} \\
\]

\[
s = e + \tau \frac{de}{dt} = (n - n_o) + \tau \frac{dn}{dt}
\]
Design and analysis.

\[
\frac{dx}{dt} = f(x, \lambda, \theta, t)
\]
\[
\dot{\theta} = u_1
\]
\[
\dot{\lambda} = u_2
\]
\[
s = (n - n_o) + \tau \frac{dn}{dt}
\]

- If \( n_0 > n \land \theta = \theta_{max} \) then \( u_1 = 0 \land u_2 = k_2 \text{sign}(s) \),
- else, \( u_1 = k_1 \text{sign}(s) \land u_2 = 0 \) (\( \lambda = \lambda_{max} \)).
The saturation limits of the present actuators are given by
\[ \lambda_{\text{min}} = 0.8 \leq \lambda \leq \lambda_{\text{max}} = 1.2, \quad \theta_{\text{min}} = -5 \leq \theta < \theta_{\text{max}} = 30; \]
and the control gains are set to \( k_1 = -100 \) and \( k_2 = 10 \). These gains can be obtained through the nominal plant parameters and the bounds of the load torque.

**ISD**

\[
\begin{align*}
\frac{dn}{dt} &= \frac{n - n_o}{\tau} \\
\frac{dp_i}{dt} &= P_i(p_i, n_o)
\end{align*}
\]
Simulation results

To illustrate the proposed input implementation, an artificial ramp load is applied to the system, ranging from 0 to 54 Nm.

- The maximum torque generated by the engine is obtained with inputs at the limit values $\theta_{max}$ and $\lambda_{min}$.
- For the engine under consideration it results $T_{e_{max}} = 52 Nm$, therefore this is the maximum admissible load torque.
To determine the feasibility of the proposed SM approach to control lean-burn IC engine, the performance and robustness of the SM control strategy has been assessed through intensive simulation tests. Given that real actuators can saturate, anti wind-up integrators are used in the simulations, and will be eventually implemented in the actual engine.
The proposed SM controller with complementary inputs is contrasted with a stoichiometric SM controller.

In the latter, the injector is commanded to maintain the air-fuel mass at stoichiometric ratio $a_s = 14.6$ (this implies constant input $\lambda = 1$), while input $\theta$ is controlled by an integrated discontinuous control action ($\theta = \int (k \text{ sign } (s)) \, dt$).
To appreciate the proposed controller performance in the presence of disturbances, in a subsequent test a 10% error has been added to the nominal engine torque (green line). It is intended to take into account uncertainties in the model parameters and/or unmeasurable perturbations.
These figures show the time evolution of the spark advance and $\lambda$ for the stoichiometric SM controller (in nominal conditions of operation) and the proposed SM controller (nominal and disturbed).

The complementary of the proposed inputs can be clearly appreciated in these figures (see blue and green lines, which correspond to the designed controller).
The success of the proposed complementary inputs strategy to attain fuel consumption reduction can be evaluated through the fuel conversion efficiency.
Conclusions

- The results presented in this paper correspond to the first stage of a broader research project which aims at developing a control system for the actual IC Engine of a novel hybrid vehicle. Specifically the design an IC Engine robust controller.

- To provide smooth control action for the actuators, the system has been expanded with integrators in the inputs. The designed SM controller has successfully fulfilled two simultaneous control objective: robust regulation of the engine speed at its nominal value and reduction of fuel consumption. The former was attained by defining the sliding surface in terms of the speed error. The latter was fulfilled through a complementary inputs implementation.
Moreover, the proposed system topology eliminates an actuator (the throttle), hence cost and complexity of the control system are substantially reduced.

Finally, it is worthwhile to mention that the fuel efficiency obtained with the proposed control strategy is better than the one obtained with the IC engine operating at fixed stoichiometric air-fuel ratio.

Future work comprises the development of the engine model of the actual hybrid power train, the design, implementation, and testing of the SM based control system in the real engine and further study of its robustness under exacting conditions.
Thank you very much for your attention