Actuator Fault-Tolerant Control based on Set Separation

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Introduction

- Fault tolerance capabilities of control loops
- Independent mechanisms
  - Fault detection
  - Fault isolation
  - Fault estimation
- Proposed approach based on set separation
- Detect the fault (from a preestablished set of fault situations) and reconfigure the system almost simultaneously
- FDI design is simple since its complexity depends linearly of the number of considered fault situations
- Approach proposed considers the generalization of a type of observers
Fault Detection and Reconfiguration Scheme

Constitutive Subsystems

- Nominal Plant and Fault Models
- Exogenous System for Reference Tracking (Exosystem)
- Plant State Observers
- State Estimate Selector
- Feedback Control Laws
- Fault Diagnosis and Isolation Module
Nominal Plant and Fault Models

- LTI perturbed system
  \[ \dot{x}(t) = Ax(t) + B'u(t) + Ed(t), \]  
  \[ y(t) = Cx(t), \]  
  with
  - \( B' \triangleq BL \),
  - \( x(t) \in \mathbb{R}^n \), \( u(t) \in \mathbb{R}^m \) and \( d(t) \in \mathbb{R}^p \) is an unknown disturbance.

**Assumption**

\[ |d(t)| \leq d_{\text{max}}, \text{ where } d_{\text{max}} \in \mathbb{R}^p \text{ is a known vector with nonnegative components.} \]

- Matrix \( L \) used to model the occurrence of actuator faults
  \[ L_0 = I, \quad L_i = \text{diag}[1 \ldots 0 \ldots 1], \quad i = 1, \ldots, m. \]

**Assumption**

The system (1)–(2) is stabilisable for all possible values of \( L = L_i \), with \( i = 0, \ldots, m \).

- Notation according to control signal \( u(t) \) and fault matrix \( L \)
  \[ \dot{x}_{i,j}(t) = Ax_{i,j}(t) + BL_iu_j(t) + Ed(t), \]
  \[ y_{i,j}(t) = Cx_{i,j}(t). \]
Exogenous System for Reference Tracking (Exosystem)

- Generation of input and state reference trajectories, $u_{ref,j}(t)$ and $x_{ref,j}(t)$, to be used under each possible fault situation

$$\dot{x}_{ref,j}(t) = Ax_{ref,j}(t) + BL_j u_{ref,j}(t), \quad (4a)$$
$$y_{ref,j}(t) = C x_{ref,j}(t), \quad (4b)$$

- $x_{ref,j}(t)$ and $u_{ref,j}(t)$ are bounded signals.
- Auxiliary control loop inside the exosystem.
- Exosystem model (4) mimics the plant model.

Assumption

The reference inputs $u_{ref,j}(t)$, for $j = 0, \ldots, m$, are assumed to be bounded and expressible as

$$u_{ref,j}(t) = \bar{u}_{ref,j} + \tilde{u}_{ref,j}(t),$$

where $\bar{u}_{ref,j} \in \mathbb{R}^m$ are constant offset levels and $\tilde{u}_{ref,j}(t)$ are variations around the respective offsets, whose amplitudes are bounded as $|\tilde{u}_{ref,j}(t)| \leq \tilde{u}_{max}^{ref,j}$, for all $t$, where $\tilde{u}_{max}^{ref,j} \in \mathbb{R}^m$ are known vectors containing nonnegative components.

- The exosystem (4) is designed such that its output $y_{ref,j}(t)$ exponentially tracks an external signal $y^*(t)$, that is,

$$\lim_{t \to \infty} [y_{ref,j}(t) - y^*(t)] = 0.$$
Plant State Observers

- Bank of observers inspired in the Unknown Input Observers (UIO).

  Wang D., Lum KY.
  *Adaptive unknown input observer approach for aircraft actuator fault detection and isolation.*

- Difference: Matrix $C$ can have rank $q < n$, i.e., full state measurement not required.

- Characteristic expression

  \[
  \dot{w}^k_{i,j}(t) = Fw^k_{i,j}(t) + GBL_ku_j(t) + My_{i,j}(t),
  \]
  \[
  \hat{x}^k_{i,j}(t) = w^k_{i,j}(t) + Hy_{i,j}(t),
  \]

  with $k = 0, \ldots, m$, $\hat{x}^k_{i,j}(t) \in \mathbb{R}^n$ the state estimate, $w^k_{i,j}(t) \in \mathbb{R}^n$ the observer state.

- Design conditions

  \[
  G = I - HC,
  \]
  \[
  F = GA - M_1C,
  \]
  \[
  M_2 = FH,
  \]
  \[
  M = M_1 + M_2,
  \]

  with $F$ a Hurwitz matrix.

- $H = 0$ corresponds to the standard Luenberger-observer-type observer.
Complementary Subsystems

- State Estimate Selector.
  - Selects a state estimate from the bank of observers.
  - Acts according to decision of FDI module.
  - Module output: state estimate from the observer \( k = j \), i.e., \( \tilde{x}_{i,j}(t) \)

- Feedback Control Laws.
  - Set of gains computed off-line for nominal and fault cases.
  - Tracking error for the selected state estimate
    \[
    \hat{z}_{i,j}(t) \triangleq \hat{x}_{i,j}(t) - x_{\text{ref},j}(t), \quad \text{for } i, j = 0, \ldots, m.
    \]
    Hence, the control expression for each scenario
    \[
    u_j(t) \triangleq K_j \hat{z}_{i,j}(t) + u_{\text{ref},j}(t).
    \]

- Fault Diagnosis and Isolation (FDI) Module. Formed by two submodules
  - Error computation module: computes the output estimation error
    \[
    e_{i,j}^k(t) \triangleq y_{i,j}(t) - \hat{y}_{i,j}^k(t),
    \]
    for \( k = 0, \ldots, m \), where \( \hat{y}_{i,j}^k(t) \triangleq C \hat{x}_{i,j}^k(t) \).
  - Decision module: implements the FDI algorithm.
Closed-Loop Dynamic Equations

- **Definitions**
  \[
  z_{i,j}(t) \triangleq x_{i,j}(t) - x_{\text{ref},j}(t), \quad \text{(state tracking error)}
  \]
  \[
  \tilde{x}_{i,j}^k(t) \triangleq x_{i,j}(t) - \hat{x}_{i,j}^k(t), \quad \text{(state estimation error)}
  \]

- **Control input**
  \[
  u_j(t) = K_j z_{i,j}(t) - K_j \zeta_{i,j}(t) + u_{\text{ref},j}(t),
  \]
  where \( \zeta_{i,j}(t) \triangleq \tilde{x}_{i,j}^j(t) \) is the state estimation error with \( k = j \).

- **If** \( k = j \), then
  \[
  \begin{bmatrix}
    \dot{z}_{i,j}(t) \\
    \dot{\zeta}_{i,j}(t)
  \end{bmatrix} = A_{i,j} \begin{bmatrix}
    z_{i,j}(t) \\
    \zeta_{i,j}(t)
  \end{bmatrix} + B_{i,j} \varphi_j(t), \quad i = 0, \ldots, m, \ j = 0, \ldots, m,
  \]
  where
  \[
  A_{i,j} \triangleq \begin{bmatrix}
    A + B L_i K_j \\
    GB(L_i - L_j) K_j \\
    F - GB(L_i - L_j) K_j
  \end{bmatrix},
  \]
  \[
  B_{i,j} \triangleq \begin{bmatrix}
    B(L_i - L_j) \\
    GB(L_i - L_j) \\
    GE
  \end{bmatrix}, \quad \varphi_j(t) \triangleq \begin{bmatrix}
    u_{\text{ref},j}(t) \\
    d(t)
  \end{bmatrix},
  \]

**Assumption**

The feedback control gains \( K_j \) and matrices \( H \) and \( M_1 \) (and hence \( G \) and \( F \)) are such that the closed-loop matrices \( A_{i,j} \), for \( i = 0, \ldots, m \) and \( j = 0, \ldots, m \), are Hurwitz.
Closed-Loop Dynamic Equations (II)

- Internal stability related to the design of $A_{i,j}$
  - Kalman Filter Design instead UIOs and gains $K_j$.
  - Iterative stability testing.
  - Solve the BMI

\[
\begin{bmatrix}
A_{i,j}^T P_{i,j} + P_{i,j} A_{i,j} & 0 \\
0 & -P_{i,j}
\end{bmatrix} < 0, \quad \text{for } i = 0, \ldots, m, \ j = 0, \ldots, m.
\]

- If $k \neq j$, then the dynamics of the state estimation errors are

\[
\dot{x}^k_{i,j}(t) = F\hat{x}^k_{i,j}(t) + N^k_{i,j} \nu_{i,j}(t), \quad i, j, k = 0, \ldots, m, \ k \neq j,
\]

with

\[
N^k_{i,j} \triangleq \begin{bmatrix}
GB(L_i - L_k)K_j & -GB(L_i - L_k)K_j & GB(L_i - L_k) & GE
\end{bmatrix},
\]

\[
\nu_{i,j}(t) \triangleq \begin{bmatrix}
z_{i,j}(t)^T & \zeta_{i,j}(t)^T & u_{\text{ref},j}(t)^T & d(t)^T
\end{bmatrix}^T.
\]

- Considering measurement noise $\eta(t) \in \mathbb{R}^q$

\[
y(t) = Cx(t) + \eta(t),
\]

then it is convenient to select $H = 0$ to avoid $\dot{\eta}(t)$ in the dynamics of the state estimation errors.
Main objective: Knowing where the output estimation errors live in
\[ e_{i,j}^k(t) \triangleq y_{i,j}(t) - \hat{y}_{i,j}^k(t), \]
for \( k = 0, \ldots, m \), where \( \hat{y}_{i,j}^k(t) \triangleq C\hat{x}_{i,j}^k(t) \).

It is necessary to know the behaviour of the state trajectories of the closed-loop
system and therefore their **ultimate bound sets**.

Variables are defined with a constant offset level and variations around their
respective offsets.

It is supposed that disturbances does not have offset level.

Performing the change of coordenates
\[
\tilde{u}_{\text{ref},j}(t) = u_{\text{ref},j}(t) - \bar{u}_{\text{ref},j},
\tilde{z}_{i,j}(t) = z_{i,j}(t) - \bar{z}_{i,j},
\tilde{\zeta}_{i,j}(t) = \zeta_{i,j}(t) - \bar{\zeta}_{i,j},
\]
then the **ultimate bound sets** for \( \tilde{z}_{i,j}(t) \) and \( \tilde{\zeta}_{i,j}(t) \) are
\[
\left| \begin{bmatrix} \tilde{z}_{i,j}(t) \\ \tilde{\zeta}_{i,j}(t) \end{bmatrix} \right| \leq |V_{i,j}| \left| \text{Re}(\Lambda_{i,j}) \right|^{-1} |V_{i,j}^{-1} B_{i,j}| \begin{bmatrix} \tilde{u}_{\text{ref},j}^{\max} \\ d^{\max}_{\text{max}} \end{bmatrix},
\]
where \((\Lambda_{i,j}, V_{i,j})\) are the matrices of the Jordan decomposition
\[ A_{i,j} = V_{i,j} \Lambda_{i,j} V_{i,j}^{-1}. \]
Therefore, the invariant sets for $\tilde{\zeta}_{i,j}(t)$ are

$$S_{i,j}^j \equiv \left\{ \zeta_{i,j} \in \mathbb{R}^n : |\zeta_{i,j} - \bar{\zeta}_{i,j}| \leq \tilde{\zeta}_{i,j}^{\max} \right\}, \quad i = 0, \ldots, m, \ j = 0, \ldots, m,$$

Similarly, the invariant sets for the state estimation errors $\tilde{x}_{i,j}^k(t)$, for $i = 0, \ldots, m, \ j = 0, \ldots, m, \ k = 0, \ldots, m, \ k \neq j$ are

$$S_{i,j}^k = \tilde{S}_{i,j}^k \oplus \{\bar{\chi}_{i,j}^k\}, \quad i, j, k = 0, \ldots, m, \ k \neq j,$$

that is the Minkowski sum of the singleton $\{\bar{\chi}_{i,j}^k\}$ and

$$\tilde{S}_{i,j}^k = \left\{ \tilde{\chi}_{i,j}^k \in \mathbb{R}^n : \left| V^{-1} \tilde{\chi}_{i,j}^k \right| \leq \left| (\text{Re}(\Lambda))^{-1} \right| \left| V^{-1} N_{i,j}^k \right| \right\}.$$

Kofman E., Haimovich H., Seron MM.

*A systematic method to obtain ultimate bounds for perturbed systems.*

The sets where the output estimation errors $e_{i,j}^k(t)$ lie whenever $\tilde{x}_{i,j}^k(t)$ belong to $S_{i,j}^k$ are computed as

$$\mathcal{E}_{i,j}^k \triangleq CS_{i,j}^k = \left\{ e_{i,j}^k \in \mathbb{R}^n : e_{i,j}^k = C\tilde{x}_{i,j}^k, \tilde{x}_{i,j}^k \in S_{i,j}^k \right\},$$

for $i = 0, \ldots, m, j = 0, \ldots, m, k = 0, \ldots, m$.

Sets related to the matching of the fault situation (index $i$) and the observer (index $k$) yield

$$S_{k,j}^k = \left\{ \tilde{x}_{k,j}^k \in \mathbb{R}^n : \left| V^{-1}\tilde{x}_{k,j}^k \right| \leq \left| (\text{Re}(\Lambda))^{-1} \right| \left| V^{-1}GE \right| d_{\text{max}} \right\},$$

and hence

$$\mathcal{E}_{k,*}^k \triangleq CS_{k,j}^k.$$

Since $d(t)$ is assumed to have no offset level, then $\mathcal{E}_{k,*}^k$ are centered at the origin.
Fault Detection Criterion

- A key property required is
  \[ \mathcal{E}_{i,j}^k \cap \mathcal{E}_{k,*}^k = \emptyset \quad \text{for} \quad i = 0, \ldots, m, \ j = 0, \ldots, m, \ k = 0, \ldots, m, \ i \neq k. \]

- It is assumed that sets can be separated by balls \( B_{r_k} \) of radii \( r_k \) centered around the origin, so
  \[
  \min \left\{ \| e_{i,j}^k \|_2 : e_{i,j}^k \in \bigcup_{i=0, i\neq k}^m \bigcup_{j=0}^m \mathcal{E}_{i,j}^k \right\} > \max \left\{ \| e_{i,j}^k \|_2 : e_{i,j}^k \in \mathcal{E}_{k,*}^k \right\},
  \]
  for each \( k = 0, \ldots, m \).

- Settling time for trajectory convergence \( t_c \). Computed/estimated by using sets \( S_{i,j}^k \) instead of sets \( \mathcal{E}_{i,j}^k \) due to the invariance condition.

FDI Criterion Algorithm

1. While \( e_{i,j}^k(t) \), for some \( k \in \{0, \ldots, m\} \), is inside the corresponding ball \( B_{r_k} \), keep control law \( K_j \) in place, with \( j = k \);
2. If \( e_{i,j}^k(t) \) leaves the corresponding ball \( B_{r_k} \), wait for \( t_c \) units of time;
3. Check all trajectories \( e_{i,j}^k(t), t \geq t_c, \) for \( k = 0, \ldots, m \). Choose the control law \( K_{\tilde{j}} \), with \( \tilde{j} = \tilde{k} \), corresponding to the trajectory \( e_{i,j}^k(t) \) that is inside the corresponding ball \( B_{r_{\tilde{k}}} \).
Illustrative Example

- Two system states
- Two possible fault scenarios

Sets for Observer 0  Sets for Observer 1  Sets for Observer 2
Offset value in the reference signal $y^*(t) \rightarrow \bar{u}_{\text{ref},j} \rightarrow \text{offset in } \mathcal{E}_{i,j}^k$.

Offset $\bar{u}_{\text{ref},j}$ should be “large enough” with respect to $d^{\text{max}}$.

Fault of the actuator $j \in \{1, \ldots, m\}$ implies a degree of freedom in vector $u_j(t) \in \mathbb{R}^m$, thus

$$u'_{\text{ref},j}(t) \triangleq u_{\text{ref},j}(t) + u_{\text{df},j},$$

where

$$u_{\text{df},j} = \begin{cases} \mathcal{X}[0 \cdots j \downarrow 1 \cdots 0]^T & \text{for fault situation} \\ [0 \cdots 0]^T & \text{for nominal situation} \end{cases}$$

Related to the latter condition, components of gains $K_j$ might be conveniently modified, taking into account the stability condition for $A_{i,j}$ (state tracking and state estimation errors).
Numerical Example

System dynamics

\[ \dot{x}_{i,j}(t) = Ax_{i,j}(t) + BL_i u_j(t) + Ed(t), \]
\[ y_{i,j}(t) = C x_{i,j}(t), \]

with

\[ A = \begin{bmatrix} -\frac{1}{R_{eq} C_p} & \frac{R_1}{R_{eq} C_p} \\ \frac{1}{L} \left( \frac{R_2}{R_{eq}} - 1 \right) & -\frac{1}{L} \left( \frac{R_1 R_2}{R_{eq}} - R_3 \right) \end{bmatrix}, \]
\[ B = \begin{bmatrix} \frac{1}{R_{eq} C_p} & 0 \\ -\frac{1}{L R_{eq}} & \frac{1}{L} \end{bmatrix}, \quad \text{and} \quad E = \begin{bmatrix} \frac{\alpha_1}{R_{eq} C_p} \\ \frac{\alpha_2 - \frac{R_2}{R_{eq}} \alpha_1}{L} \end{bmatrix}. \]
The capacitor voltage is required to track a reference signal

\[ y^*(t) = a + b \sin \omega t, \]

where \( \omega = 20\pi \), \( a = 50V \) and \( b = 1.5V \).

Considered fault scenarios

- **Scenario 0**: Both voltage sources, \( V_1 \) and \( V_2 \), are operational. This scenario is modeled by \( L_i = L_0 \), where

  \[
  L_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
  \]

- **Scenario 1**: Voltage source \( V_1 \) is short-circuited, that is \( V_1(t) = 0 \), and \( V_2 \) is operational. This fault scenario is modeled by \( L_i = L_1 \), where

  \[
  L_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.
  \]

- **Scenario 2**: Voltage source \( V_2 \) is short-circuited, that is \( V_2(t) = 0 \), and \( V_1 \) is operational. This fault scenario is modeled by \( L_i = L_2 \), where

  \[
  L_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.
  \]

Feedback control gains \( K_j \) designed using LQR methodology (Riccati equation).
Sequence of simulated fault scenarios (top graphic) and the corresponding FDI decision (bottom graphic).
Voltage signals. Continuous line: reference $y^*(t)$, dashed line: estimated capacitor voltage $\hat{v}_C(t)$ (at the output of the SES), dotted line: measured capacitor voltage $v_C(t)$. 
Sets and balls \( B_{r_k} \) for the observers related to the electric circuit.

Sets for Observer 0

Sets for Observer 1

Sets for Observer 2
Conclusions and Current/Future Work

- Actuator fault-tolerant approach based on a bank of observers type UIO.
- Discrete number of fault scenarios previously defined.
- Easily checkable set of conditions for FDI and closed-loop stability.
- Computation of sets where system trajectories lie.
- Approach conditioned by offset levels at reference signals.
- Particular applications.
- Different feedback stabilizing control laws.
- Consideration of disturbances with offset levels.
- Alternative topologies of double-observer evaluation.
Thank you very much and...

Força Barça!!